Lognormal Distributions for Body Weight as a Function of Age for Males and Females in the United States, 1976 - 1980

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Abstract

Based on results reported from the NHANES II Survey (the National Health and Nutrition Examination Survey II) for people living in the United States during 1976 - 1980, we use exploratory data analysis, probability plots, and the method of maximum likelihood to fit Lognormal distributions to percentiles of body weight for males and females as a function of age from 6 months through 74 years. The results are immediately useful in probabilistic (and deterministic) risk assessments.

1.0 Introduction

To conduct a probabilistic risk assessment that includes people of different ages in an exposed population, a risk assessor needs distributions for the body weights of children, teens, and adults as a function of age. While the US EPA has published a variety of summary statistics for body weights (e.g., US EPA, 1989, EFH; US EPA, 1995, EFH2), no one has published parametric distributions for body weight as a function of age. We use the data reported by the NHANES II Survey (NCHS, 1987) to fit probability distributions for use in probabilistic risk assessments.

2.0 NHANES II Survey

The NHANES II Survey collected information on the nutritional status and related factors to estimate the prevalence of overweight people in the United States non-institutionalized population (NCHS, 1987). Conducted by the National Center for Health Statistics (NCHS) from February 1976 through February 1980, the study focused on civilians from 6 months through 74 years of age in 50 states.

The entire NHANES II sample included 27,801 persons. The body weights of 10,295 males and 10,667 females were collected and reported after the data were statistically adjusted for nonresponse and probability of selection and then post-stratified by age, sex, and race to reflect the whole US population (Exhibit 1; NCHS, 1987). The weight measurements were to the nearest 0.25 kg and included clothing (estimated as ranging from 0.09 to 0.28 kg).

The survey used a stratified, multi-stage design (NCHS, 1987) that selected samples at each stage with a known probability of sampling males and females. In hierarchical order, the stages of selection were: primary sampling units (PSUs), which are counties or small groups of contiguous counties; census enumeration districts; segments (clusters of households); households; and sample persons. Approximately one person per sample household was selected, with younger and older age groups over-sampled.

The NCHS derived national probability estimates through a multi-stage estimation procedure with three main steps: (i) inflation by the reciprocal of the probability of selection, (ii) adjustment for nonresponse, and (iii) post-stratification by age, sex, and race. (NCHS, 1987, Appendix I). The probability of selection is the product of probabilities from each stage of selection in the design - PSU, segment, household, and sample person. To adjust for nonresponse, the estimates were inflated by a factor that increased estimates based on examined persons to the value that would have been achieved if all sample persons had been examined. To post-stratify by age, sex, and race, estimates of the number of examined persons were ratio adjusted within each of 75 age-sex-race cells to independent estimates, provided by the US Bureau of the Census, of the population for 1 March 1978, the approximate midpoint of the survey. The ratio adjustment used a factor in which the numerator was the US population and the denominator was the sum of the weights adjusted for nonresponse for examined persons. This adjustment brought the population estimates into close agreement with the US Bureau of the Census estimates of the civilian non-institutionalized population.

3.0 Data

The NCHS reported the NHANES II results as summary statistics for the body weights of males and females of different ages. Tables 1M and 1F, respectively, show these data for males and females: the age group, the number, the arithmetic mean (AMean), the arithmetic standard deviation (AStdDev), and the 5th, 10th, 15th, 25th, 50th, 75th, 85th, 90th, and 95th percentiles of body weight measured in kg.

4.0 Exploratory Data Analysis

During the exploratory data analysis (Chambers et al, 1983; Cleveland, 1985; Tukey, 1977), we fit the Lognormal distribution (Evans et al, 1993) to the data using Lognormal probability plots (D'Agostino & Stephens, 1986):

$$\ln[w] \sim \text{Normal}(\mu, \sigma) \qquad \text{Eqn 1}$$

where w represents body weight in kg, $\ln[\bullet]$ represents the natural logarithm function, $\text{Normal}[\bullet, \bullet]$ represents the Normal or Gaussian probability distribution with parameters μ (mean) and σ (standard deviation).

For each age group and sex, we found that the probability data plotted in reasonably straight lines on Lognormal probability plots. We concluded that Lognormal distributions would give reasonable fits to the data for each sex across all age groups, a finding consistent with results previously published for adult men and women in the United States (Brainard & Burmaster, 1992).

The left side of Table 2 shows the best estimates for $\hat{\mu}$ and $\hat{\sigma}$ (and adjusted R^2 values, denoted $adjR^2$) from the linear regressions on the probability plots. Note that all the $adjR^2 > 0.95$. Figures 1M and 1F, respectively, show the probability plots with the lowest $adjR^2$ values -- age 6 to 7 yr for males and age 9 to 10 yr for females.

5.0 Maximum Likelihood Estimation (MLE)

We used the method of maximum likelihood estimation (MLE; Edwards, 1992) to fit Lognormal distributions to the data. We start with a more generalized theoretical development than is necessary (Crouch et al, 1996). Label the age groups by i, $i = 1, 2, \ldots, N$, with n_i people measured in age group i, with the central age of the age group being t_i in years; and let the j^{th} weight percentile in age group i be at probability p_{ij}, with

$$0 = p_{i0} < p_{i1} < \ldots < p_{iM_i} < p_{iM_i+1} = 1 \qquad \text{Eqn 2}$$

so that the M_i given percentiles at age i are p_{i1} through p_{iM_i}, and these are augmented for convenience with 0 and 1 at the low and high ends. Let the value at the j^{th} percentile in age group i be at weight w_{ij}, with

$$0 = w_{i0} < w_{i1} < ... < w_{iMi} < w_{iM_i+1} = \infty \qquad \text{Eqn 3}$$

where the bottom and top values are appended for convenience of notation. When we take natural logarithms of the weights, we assume $\ln[0] = -\infty$ and $\ln[\infty] = \infty$.

5.1 MLE for Each Age Group and for Each Gender

Assume that $\ln[w]$ for each age group i follows a Normal distribution, with mean μ_i and standard deviation σ_i. If all sampling had been at random from the population, the loglikelihood function (J_i) for the data for each gender for each age group i would be:

$$J_i = \sum_{j=0}^{M_i} n_i \bullet (p_{ij+1} - p_{ij}) \bullet \ln\left[\Phi\left(\frac{\ln[w_{ij+1}] - \mu_i}{\sigma_i} \right) - \Phi\left(\frac{\ln[w_{ij}] - \mu_i}{\sigma_i} \right) \right]$$

Eqn 4

where $\Phi(\bullet)$ is the integral of the standard (unit) Normal distribution. In fact, the sample was designed as a stratified random sample, with the observations weighted according to the sample design. However, we do not know the sampling weights, nor do we know how the distributions may differ in the various strata. In the absence of such information, we use Eqn 4 as a suitable approximation to the loglikelihood function for each age group for each sex.

The right side of Table 2 shows the MLE estimates $\hat{\mu}$ and $\hat{\sigma}$ for each age group and each sex. The table also reports the maximum of the loglikelihood function (MaxLL) for each age group and each gender, and the table also reports the maximum of the loglikelihood function divided by the number of the people in each group (MaxLL/n).

5.2 Parametric Estimates for $\hat{\mu}$ and $\hat{\sigma}$

If we assume that μ and σ are continuous functions of age, then with the same assumptions as before the loglikelihood function for all ages combined is:

$$J = \sum_{i=1}^{N} \sum_{j=0}^{M_i} n_i \cdot (p_{ij+1} - p_{ij}) \cdot \ln\left[\Phi\left(\frac{\ln[w_{ij+1}] - \mu_i}{\sigma_i} \right) - \Phi\left(\frac{\ln[w_{ij}] - \mu_i}{\sigma_i} \right) \right]$$

Eqn 5

We parameterize $\mu(t)$ and $\sigma(t)$ as follows:

$$\mu(t) = ma_0 + ma_1 \cdot t + ma_2 \cdot t^2 + mb \cdot \exp[-t \cdot (mc_0 + mc_1 \cdot t + mc_2 \cdot t^2)]$$

Eqn 6

$$\sigma(t) = sa + (sb_0 + sb_1 \cdot t + sb_2 \cdot t^2 + sb_3 \cdot t^3 + sb_4 \cdot t^4 + sb_5 \cdot t^5) \cdot \exp[-(sc \cdot t)^4]$$

Eqn 7

There is no theoretical justification for the selection of a model of this form. We chose Eqns 6 and 7 because they provide a parsimonious fit to the reported data. We set the power in the exponential in Eqn 7 to 4 as a compromise between the best integer values for males (7 or 8) and for females (2).

Table 3 shows the best-fit values for the 15 parameters for males and females. [EndNote 1]. Figures 2M and 2F depict $\mu(t)$ from the MLE method as a line and μ_i from Table 2 as points. Figures 3M and 3F depict the corresponding results for $\sigma(t)$; the results align well for males and adequately for females. In Figures 4M and 4F, the lines depict the arithmetic mean of $w(t)$ along with the 5th and the 95th percentiles of $w(t)$ as fit by MLE as plotted while the points depict the corresponding data from Table 1. For males, agreement is close. For females, these fits underestimate the points from puberty through age ~55 yr, demonstrating a limitation of the results. For the 95th percentile of $w(t)$ for females, the maximum difference (~5 kg) between the curve fit by MLE and the data in Table 1F occurs between 35 and 44 years of age. From further analysis, we know that the lines and points in Figure 4F for females diverge near puberty and then converge again because Eqn 1 cannot capture the non-Lognormal behavior of the data in Table 1F (see also the discussion in Brainard & Burmaster, 1992).

6.0 Discussion

Balancing the competing objectives of goodness-of-fit with parsimony of representation, we conclude that the model in Eqns 5, 6, and 7 and the best-fit parameters for males and females in Table 3 are appropriate for use in probabilistic exposure assessments. Although these equations and best-fit parameters do not provide a perfect fit to the NHANES II data for 1976 - 1980, the errors are small. In most risk assessment applications, the errors lead to a slight bias in a health-protective direction (because the fitted curves tend to underestimate some of the high percentiles of w(t), a variable that usually occurs in the denominator of a dose equation).

EndNotes

1. EACC obtained these estimates using the optimizer in Borland QuattroPro™ 5.0 to maximize the loglikelihood function. The cumulative Normal integral Φ was implemented in a custom add-in function. DEB plotted the graphs in Mathematica™ 2.2.

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Table 1M

Body Weights of Males

Age (yr)	Mid (yr)	Number (n)	AMean (kg)	AStdDev (kg)	5th Percentile (kg)	10th Percentile (kg)	15th Percentile (kg)	25th Percentile (kg)	50th Percentile (kg)	75th Percentile (kg)	85th Percentile (kg)	90th Percentile (kg)	95th Percentile (kg)
0.5 to 1	0.75	179	9.4	1.3	7.5	7.6	8.2	8.6	9.4	10.1	10.7	10.9	11.4
1 to 2	1.50	370	11.8	1.9	9.6	10.0	10.3	10.8	11.7	12.6	13.1	13.6	14.4
2 to 3	2.50	375	13.6	1.7	11.1	11.6	11.8	12.6	13.5	14.5	15.2	15.8	16.5
3 to 4	3.50	418	15.7	2.0	12.9	13.5	13.9	14.4	15.4	16.8	17.4	17.9	19.1
4 to 5	4.50	404	17.8	2.5	14.1	15.0	15.3	16.0	17.6	19.0	19.9	20.9	22.2
5 to 6	5.50	397	19.8	3.0	16.0	16.8	17.1	17.7	19.4	21.3	22.9	23.7	25.4
6 to 7	6.50	133	23.0	4.0	18.6	19.2	19.8	20.3	22.0	24.1	26.4	28.3	30.1
7 to 8	7.50	148	25.1	3.9	19.7	20.8	21.2	22.2	24.8	26.9	28.2	29.6	33.9
8 to 9	8.50	147	28.2	6.2	20.4	22.7	23.6	24.6	27.5	29.9	33.0	35.5	39.1
9 to 10	9.50	145	31.1	6.3	24.0	25.6	26.0	27.1	30.2	33.0	35.4	38.6	43.1
10 to 11	10.50	157	36.4	7.7	27.2	28.2	29.6	31.4	34.8	39.2	43.5	46.3	53.4
11 to 12	11.50	155	40.3	10.1	26.8	28.8	31.8	33.5	37.3	46.4	52.0	57.0	61.0
12 to 13	12.50	145	44.2	10.1	30.7	32.5	35.4	37.8	42.5	48.8	52.6	58.9	67.5
13 to 14	13.50	173	49.9	12.3	35.4	37.0	38.3	40.1	48.4	56.3	59.8	64.2	69.9
14 to 15	14.50	186	57.1	11.0	41.0	44.5	46.4	49.8	56.4	63.3	66.1	68.9	77.0
15 to 16	15.50	184	61.0	11.0	46.2	49.1	50.6	54.2	60.1	64.9	68.7	72.8	81.3
16 to 17	16.50	178	67.1	12.4	51.4	54.3	56.1	58.7	64.4	73.6	78.1	82.2	91.2
17 to 18	17.50	173	66.7	11.5	50.7	53.4	54.8	58.7	65.8	72.0	76.8	82.3	88.9
18 to 19	18.50	164	71.1	12.7	54.1	56.6	60.3	61.9	70.4	76.6	80.0	83.5	95.3
19 to 20	19.50	148	71.7	11.6	55.9	57.9	60.5	63.8	69.5	77.9	84.3	86.8	92.1
18 thru 24	21.50	988	73.8	12.7	56.8	60.4	61.9	64.8	72.0	80.3	85.1	90.4	99.5
25 thru 34	30.00	1,067	78.7	13.7	59.5	62.9	65.4	69.3	77.5	85.6	91.1	95.1	102.7
35 thru 44	40.00	745	80.9	13.4	59.7	65.1	67.7	72.1	79.9	88.1	94.8	98.8	104.3
45 thru 54	50.00	690	80.9	13.6	60.8	65.2	67.2	71.7	79.0	89.4	94.5	99.5	105.3
55 thru 64	60.00	1,227	78.8	12.8	59.9	63.8	66.4	70.2	77.7	85.6	90.5	94.7	102.3
65 thru 74	70.00	1,199	74.8	12.8	54.4	58.5	61.2	66.1	74.2	82.7	87.9	91.2	96.6

Source: NCHS (1987) Table 6 and Table 7 (all races)
Note: Includes weight of clothing (estimated as ranging from 0.09 to 0.28 kg)

Table 1F

Body Weights of Females

Age (yr)	Mid (yr)	Number (n)	AMean (kg)	AStdDev (kg)	5th Percentile (kg)	10th Percentile (kg)	15th Percentile (kg)	25th Percentile (kg)	50th Percentile (kg)	75th Percentile (kg)	85th Percentile (kg)	90th Percentile (kg)	95th Percentile (kg)
0.5 to 1	0.75	177	8.8	1.2	6.6	7.3	7.5	7.9	8.9	9.4	10.1	10.4	10.9
1 to 2	1.50	336	10.8	1.4	8.8	9.1	9.4	9.9	10.7	11.7	12.4	12.7	13.4
2 to 3	2.50	336	13.0	1.5	10.8	11.2	11.6	12.0	12.7	13.8	14.5	14.9	15.9
3 to 4	3.50	366	14.9	2.1	11.7	12.3	12.9	13.4	14.7	16.1	17.0	17.4	18.4
4 to 5	4.50	396	17.0	2.4	13.7	14.3	14.5	15.2	16.7	18.4	19.3	20.2	21.1
5 to 6	5.50	364	19.6	3.3	15.3	16.1	16.7	17.2	19.0	21.2	22.8	24.7	26.6
6 to 7	6.50	135	22.1	4.0	17.0	17.8	18.6	19.3	21.3	23.8	26.6	28.9	29.6
7 to 8	7.50	157	24.7	5.0	19.2	19.5	19.8	21.4	23.8	27.1	28.7	30.3	34.0
8 to 9	8.50	123	27.9	5.7	21.4	22.3	23.3	24.4	27.5	30.2	31.3	33.2	36.5
9 to 10	9.50	149	31.9	8.4	22.9	25.0	25.8	27.0	29.7	33.6	39.3	43.3	48.4
10 to 11	10.50	136	36.1	8.0	25.7	27.5	29.0	31.0	34.5	39.5	44.2	45.8	49.6
11 to 12	11.50	140	41.8	10.9	29.8	30.3	31.3	33.9	40.3	45.8	51.0	56.6	60.0
12 to 13	12.50	147	46.4	10.1	32.3	35.0	36.7	39.1	45.4	52.6	58.0	60.5	64.3
13 to 14	13.50	162	50.9	11.8	35.4	39.0	40.3	44.1	49.0	55.2	60.9	66.4	76.3
14 to 15	14.50	178	54.8	11.1	40.3	42.8	43.7	47.4	53.1	60.3	65.7	67.6	75.2
15 to 16	15.50	145	55.1	9.8	44.0	45.1	46.5	48.2	53.3	59.6	62.2	65.5	76.6
16 to 17	16.50	170	58.1	10.1	44.1	47.3	48.9	51.3	55.6	62.5	68.9	73.3	76.8
17 to 18	17.50	134	59.6	11.4	44.5	48.9	50.5	52.2	58.4	63.4	68.4	71.6	81.8
18 to 19	18.50	170	59.0	11.1	45.3	49.5	50.8	52.8	56.4	63.0	66.0	70.1	78.0
19 to 20	19.50	158	60.2	11.0	48.5	49.7	51.7	53.9	57.1	64.4	70.7	74.8	78.1
18 thru 24	21.50	1,066	60.6	11.9	46.6	49.1	50.6	53.2	58.0	65.0	70.4	75.3	82.9
25 thru 34	30.00	1,170	64.2	15.0	47.4	49.6	51.4	54.3	60.9	69.6	78.4	84.1	93.5
35 thru 44	40.00	844	67.1	15.2	49.2	52.0	53.3	56.9	63.4	73.9	81.7	87.5	98.9
45 thru 54	50.00	763	68.0	15.3	48.5	51.3	53.3	57.3	65.5	75.7	82.1	87.6	96.0
55 thru 64	60.00	1,329	67.9	14.7	48.6	51.3	54.1	57.3	65.2	75.3	82.3	87.5	95.1
65 thru 74	70.00	1,416	66.6	13.8	47.1	50.8	53.2	57.4	64.8	73.8	79.8	84.4	91.3

Source: NCHS (1987) Table 6 and Table 8 (all races)

Note: Includes weight of clothing (estimated as ranging from 0.09 to 0.28 kg)

Table 2
Best-Fit Parameters for LogNormal Distributions

Age Mid-Point (yr)	Males (n)	Females (n)	LNPP Males muhat	LNPP Males sigmahat	LNPP Males adjR2	LNPP Females muhat	LNPP Females sigmahat	LNPP Females adjR2	MLE Males muhat	MLE Males sigmahat	MLE Males MaxLL	MLE Males MaxLL/n	MLE Females muhat	MLE Females sigmahat	MLE Females MaxLL	MLE Females MaxLL/n
0.75	179	177	2.22720	0.13060	0.991	2.15851	0.14502	0.989	2.23173	0.12922	-379.6	-2.121	2.16300	0.144959	-374.9	-2.118
1.5	370	336	2.45769	0.12032	0.998	2.37785	0.12920	0.998	2.45778	0.12001	-761.7	-2.059	2.37602	0.12877	-693.0	-2.062
2.5	375	336	2.60236	0.11955	0.997	2.55994	0.11294	0.992	2.60259	0.11843	-778.7	-2.076	2.55520	0.11287	-698.4	-2.078
3.5	418	366	2.74508	0.11433	0.996	2.68782	0.13605	0.999	2.74274	0.11483	-863.7	-2.066	2.68791	0.13614	-755.1	-2.063
4.5	404	396	2.86640	0.13273	0.996	2.82399	0.13428	0.996	2.86471	0.13278	-836.4	-2.070	2.82040	0.13495	-820.7	-2.073
5.5	397	364	2.98466	0.13857	0.990	2.97555	0.16351	0.981	2.97656	0.13951	-826.7	-2.082	2.93160	0.16435	-764.9	-2.101
6.5	133	135	3.12982	0.14487	0.967	3.09518	0.17402	0.980	3.11429	0.14589	-284.0	-2.135	3.08062	0.17318	-287.6	-2.130
7.5	148	157	3.21368	0.15061	0.974	3.19311	0.17430	0.980	3.20886	0.15202	-311.9	-2.107	3.18558	0.17561	-334.2	-2.128
8.5	147	123	3.32734	0.18056	0.981	3.30850	0.15637	0.992	3.31836	0.17999	-310.8	-2.114	3.30795	0.15696	-256.5	-2.085
9.5	145	149	3.42957	0.16522	0.969	3.45786	0.21427	0.958	3.41751	0.16650	-308.2	-2.125	3.43201	0.21603	-322.0	-2.161
10.5	157	136	3.58679	0.19514	0.974	3.56625	0.19869	0.996	3.57275	0.19542	-331.0	-2.108	3.55883	0.19772	-282.2	-2.075
11.5	155	140	3.68681	0.25158	0.988	3.70617	0.22638	0.984	3.67049	0.25174	-328.6	-2.120	3.69659	0.22591	-297.9	-2.128
12.5	145	147	3.77694	0.22466	0.981	3.82269	0.21339	0.999	3.76741	0.22351	-305.0	-2.103	3.81946	0.21388	-302.9	-2.061
13.5	173	162	3.88052	0.21405	0.991	3.91727	0.21478	0.981	3.87601	0.21737	-362.1	-2.093	3.90747	0.21370	-340.8	-2.104
14.5	186	178	4.02248	0.18083	0.995	3.98658	0.18690	0.995	4.02566	1.81627	-385.3	-2.071	3.98195	0.18709	-370.8	-2.083
15.5	184	145	4.09199	0.15933	0.986	4.00250	0.15610	0.958	4.09044	0.15867	-384.6	-2.090	3.99219	0.15902	-309.1	-2.132
16.5	178	170	4.19752	0.16760	0.988	4.05469	0.16696	0.988	4.18817	0.16912	-370.5	-2.081	4.04296	0.16645	-356.3	-2.096
17.5	173	134	4.18692	0.16723	0.993	4.07554	0.16468	0.977	4.18375	0.16642	-359.2	-2.077	4.06856	0.16559	-283.5	-2.116
18.5	164	170	4.24538	0.15878	0.982	4.06462	0.14720	0.975	4.24448	0.15989	-348.7	-2.126	4.05561	0.14849	-360.4	-2.120
19.5	148	158	4.26203	0.15473	0.996	4.09570	0.14906	0.975	4.25659	0.15429	-306.9	-2.074	4.08099	0.14938	-338.6	-2.143
											0.0					
21.5	988	1,066	4.29422	0.16279	0.989	4.09573	0.16785	0.981	4.28691	0.16373	-2,051.7	-2.077	4.08354	0.16833	-2,229.7	-2.092
30	1,067	1,170	4.34946	0.16280	0.999	4.15340	0.20411	0.979	4.34844	0.16256	-2,194.1	-2.056	4.13655	0.20493	-2,455.5	-2.099
40	745	844	4.37886	0.16475	0.997	4.19837	0.20711	0.980	4.37907	0.16387	-1,536.5	-2.062	4.18233	0.20872	-1,770.8	-2.098
50	690	763	4.38148	0.16567	0.999	4.19971	0.20796	0.996	4.37909	0.16565	-1,422.3	-2.061	4.19328	0.20820	-1,572.8	-2.061
60	1,227	1,329	4.35355	0.15665	0.998	4.19959	0.20483	0.996	4.35270	0.15631	-2,525.0	-2.058	4.19188	0.20490	-2,742.7	-2.064
70	1,199	1,416	4.29472	0.17355	0.998	4.17846	0.19815	0.999	4.29908	0.17322	-2,467.2	-2.058	4.17631	0.19741	-2,910.5	-2.055

Table 3
Best-Fit Parameters
for Equations 5, 6, and 7

	Males	Females
ma0	4.06661	3.90383
ma1	1.37603 E-2	1.06805 E-2
ma2	- 1.49397 E-4	- 9.72316 E-5
mb	- 2.00685	- 1.97114
mc0	1.47094 E-1	1.84877 E-1
mc1	- 1.39216 E-2	- 2.04753 E-2
mc2	9.77090 E-4	1.59966 E-3
sa	1.63829 E-1	2.03999 E-1
sb0	- 6.97297 E-3	- 2.12621 E-2
sb1	- 2.75757 E-2	- 6.50109 E-2
sb2	4.34040 E-5	2.25515 E-2
sb3	2.86364 E-3	- 3.00811 E-3
sb4	- 5.31960 E-4	1.89083 E-4
sb5	2.83428 E-5	- 4.65941 E-6
sc	1.01888 E-1	6.57724 E-2
MaxLL	21,382	22,260

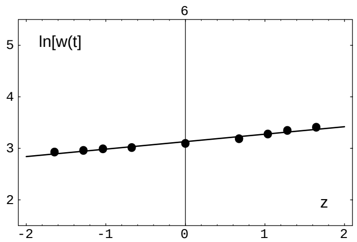

Figure 1M: Lognormal Probability Plot for Males
Age 6 to 7 yr

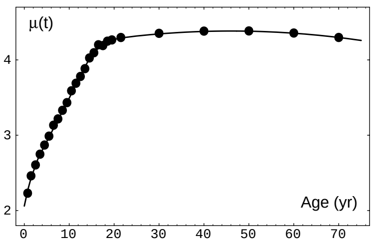

Figure 2M: Plot of μ(t) from MaxLikelihood (line) vs μ(t) from LNPPs (points) for Males

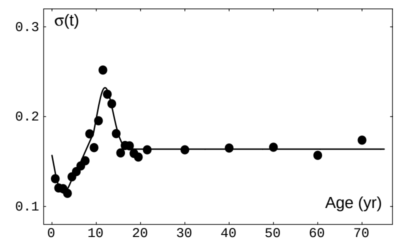

Figure 3M: Plot of σ(t) from MaxLikelihood (line) vs σ(t) from LNPPs (points) for Males

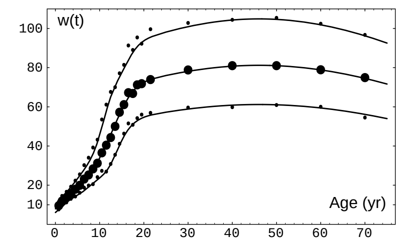

Figure 4M: Plot of AMean(w(t)) with 5th and 95th Percentiles of w(t) from MaxLikelihood vs Data from Table 1M for Males

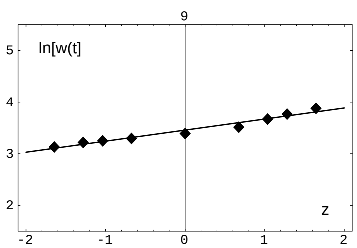

Figure 1F: Lognormal Probability Plot for Females
Age 9 to 10 yr

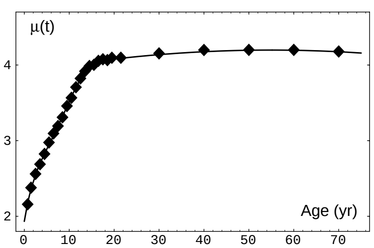

Figure 2F: Plot of μ(t) from MaxLikelihood (line) vs μ(t) from LNPPs (points) for Females

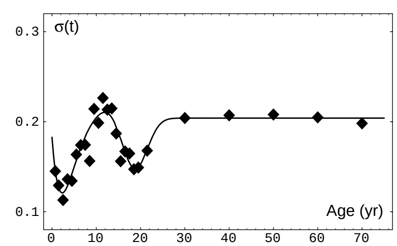

Figure 3F: Plot of σ(t) from MaxLikelihood (line) vs σ(t) from LNPPs (points) for Females

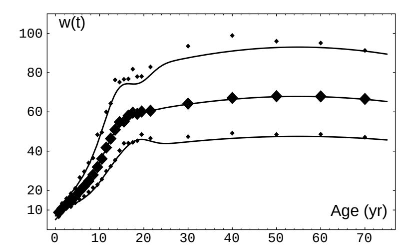

Figure 3F: Plot of AMean(w(t)) with 5th and 95th Percentiles of w(t) from MaxLikelihood vs Data from Table 1M for Females

Figures 1, 2, 3, and 4